

# White paper

## Ellipse – Performance in AHRS mode



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# 1. Abstract

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Offering reliable roll, pitch, and yaw estimations using MEMS-based inertial sensors presents significant challenges due to inherent sensor bias: their nature of drifting over time, and performance degradation during high dynamic conditions. Additionally, magnetometers, which is inherent for AHRS application for heading estimation, are highly susceptible to disturbances from surrounding environments, making reliable heading determination even more complex. This paper addresses these challenges by expanding on state-of-the-art sensor fusion techniques and introducing an advanced decision-making algorithm designed to maintain precise roll, pitch and heading estimations even in dynamic scenarios and temporarily disturbed magnetic environments. The performance of this algorithm is evaluated on the Ellipse-A, a MEMS-based AHRS, across various real-world kinematic scenarios, from static to highly dynamic environments. Field tests were conducted, and results were benchmarked against a high tactical-grade sensor for validation.

# 2. Introduction

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In today's advanced navigation and control systems, achieving and maintaining a precise orientation with MEMS based inertial measurement unit is a continuous challenge. MEMS gyroscopes alone are subject to drift over time, which requires their fusion with accelerometers to limit the drift in roll and pitch. However, even this combination cannot provide complete orientation, as heading information is still missing. External references – such as magnetometers or GNSS – are required to resolve the full orientation solution.

This paper evaluates an Attitude and Heading Reference System (AHRS) algorithm that fuses IMU and magnetometer data. The performance analysis focuses on the Ellipse-A, a MEMS-based IMU with an integrated magnetometer, developed by SBG Systems.

The key challenge addressed is delivering a robust and reliable heading solution, particularly in environments with magnetic disturbances and dynamic motion, which can significantly affect sensor accuracy and introduce errors in roll, pitch, and yaw estimates.

The paper is structured as follows:

- The first section introduces the Ellipse-A
- The second section explains the challenges of MEMS sensors and the sensor fusion algorithm used
- The final section describes the tests and analyzes the results

### 3. Product presentation

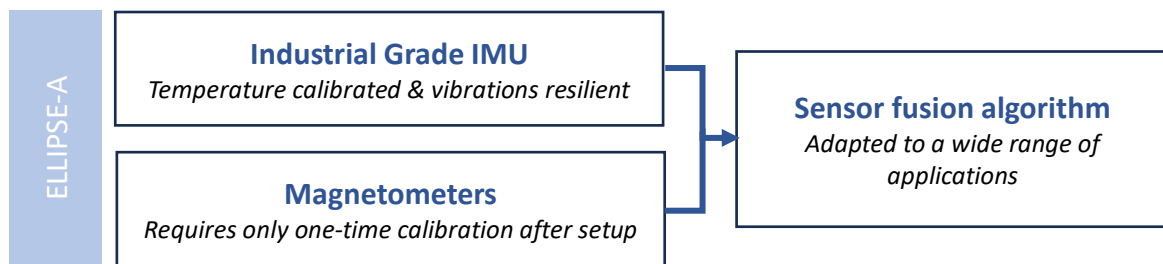
The system under evaluation is the Ellipse, a compact MEMS based product line, ranging from IMUs to GNSS-aided INS. This paper will focus on the Ellipse-A, the AHRS variant of the Ellipse series.

The Ellipse-A is composed of:

- Three-axis accelerometer to measure acceleration,
- Three-axis gyroscope to capture rotational rates, and
- Three-axis magnetometer to sense the Earth magnetic field intensity.

The three sensors are fused together within an advanced sensor fusion algorithm to estimate roll, pitch and yaw angles.

The diagram below illustrates the functional structure of the Ellipse-A.



The main specifications of the Ellipse-A are listed below:

	Accelerometers	Gyroscopes	Magnetometers
Range	Up to 40g	Up to 1000°/s	50 Gauss
In run bias instability	14 ug	7°/h	1.5 mGauss
Bias repeatability	1.5 mg	500°/h	
Bandwidth	390 Hz	133 Hz	22 Hz

## 4. Technology overview

### 4.1. Gyroscope use and error model

The gyroscope is a sensor that measures an angular rate. When integrated over time, the angular rate can be used to track the orientation, by measuring all small orientation changes increments. This sensor error model can be summarized as follows:

$$g_b = \omega + b + \rho$$

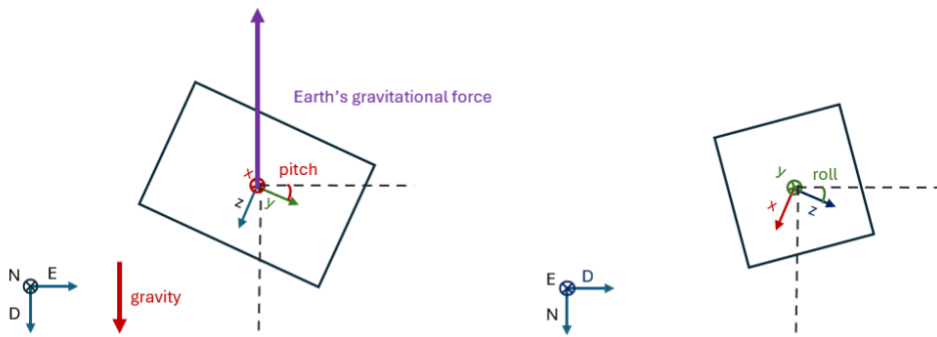
Where

- $g_b$ : measured gyro rate in body axes
- $\omega$ : True angular rate in body frame
- $b$ : gyroscope bias, slowly varying
- $\rho$ : gyroscope white noise

**Note:** In a typical Vertical gyroscope or AHRS application, the Earth’s rotation is neglected since the actual position of the sensor is not known. An INS system will take Earth rotation into account for improved performance.

### 4.2. Accelerometer use and error model

An accelerometer is a sensor that measures the specific force, which includes not only the motion induced forces, but also the constant force of gravity. When the body is stationary, a perfect accelerometer affixed to it measures the Earth's gravitational force as a constant acceleration of  $\sim 9.81 \text{ m/s}^2$  (1g) along the axis aligned with gravity. This characteristic allows for the alignment of the body frame with the sensor one, and then the resolution of the body’s orientation—roll and pitch—relative to the horizontal plane.



However, the static assumption, is rarely met, and some accelerations will add transient errors on the accelerometer measurement. This should be handled with care by the sensor fusion algorithm. In addition, some inevitable errors to the MEMS sensors affect the measurement of the gravity by adding or subtracting a constant value on the accelerometers readings.

Majority of these errors can be calibrated at our facility, thanks to a rigorous calibration procedure. However a remaining bias residual can still affect the roll and pitch performance. This is usually referred as startup bias or bias repeatability. If the other sources of bias are neglected, the accelerometer measurement in static can be expressed as:

$$a_m = g + b_r$$

Where:

- $a_m$ : measured acceleration along the axis aligned with gravity
- $g = 9.81$  m/s: Earth's gravitational force
- $b_r$ : bias repeatability error

This bias will directly contribute to roll and pitch error budget, which can be approximated by the following formula:

$$\varepsilon = \tan^{-1} \frac{b_r}{g} \approx \frac{b_r}{g}$$

Where:

- $\varepsilon$  : error in roll/pitch angles

For instance, a bias repeatability of 1.5 mg leads to an attitude error of about 0.09°. This bias cannot be calibrated at the factory, as it changes with each power cycle.

In a GNSS-aided INS, accelerometer data is continuously compared to GNSS-derived accelerations to estimate and correct this bias. However, in an AHRS, there is no external reference, so the bias cannot be estimated, and its effect remains in the roll and pitch calculations.

### 4.3. Heading determination with Magnetometers

There are several heading determination methods. In this paper, we focus on the Ellipse series, with its MEMS-based IMU and integrated magnetometer. Given this configuration, heading estimation relies on a combination of gyroscope and magnetometer data, making it essential to understand the limitations and challenges associated with this approach.

Magnetometers provide a direct measurement of the local magnetic field, which can be used to determine a magnetic heading after a dedicated magnetic calibration procedure and a tilt compensation.

#### 4.3.1. Magnetometers calibration

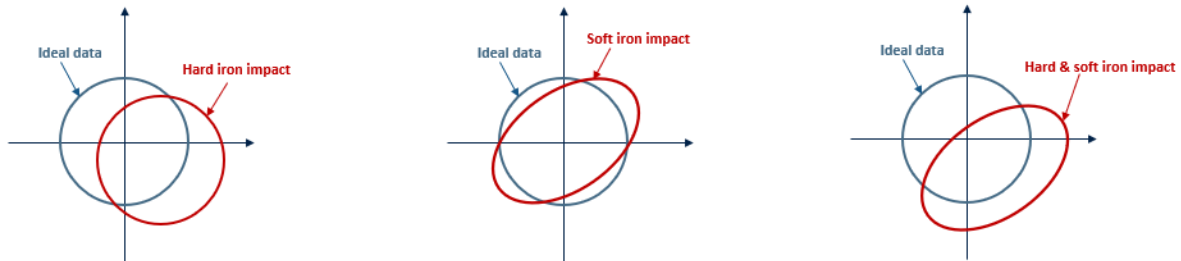
Magnetometers are very sensitive to their close environment, mainly the objects on which they are strapped. Some materials can generate magnetic fields that will be summed with Earth magnetic field, and some other can distort the existing magnetic field. To minimize the impact of the errors on the heading, a proper calibration after setup is required.

The calibration process at SBG Systems involves:

- Hard Iron compensation: corrects for distortion caused by magnets or magnetic materials, which create a constant offset in magnetic field readings.

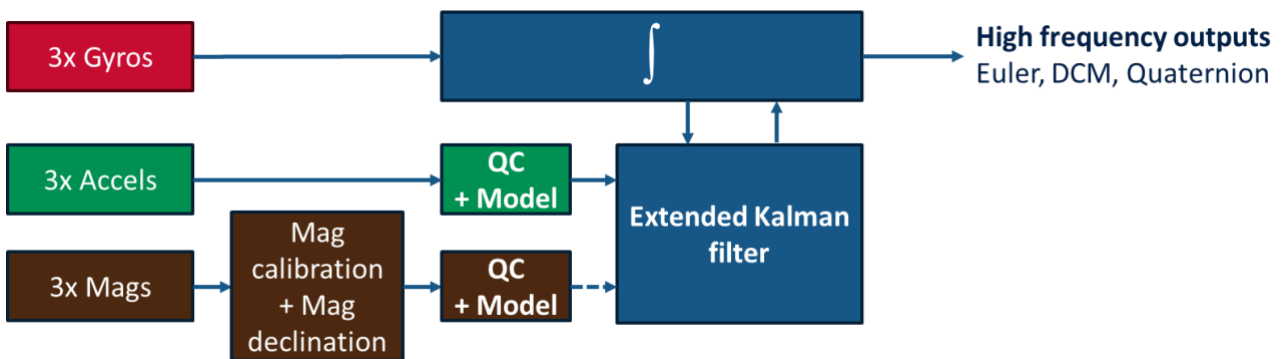
- Soft Iron compensation: corrects for distortion caused by ferromagnetic materials that interact with existing magnetic fields, altering both the field strength and direction, making the compensation more challenging.
- Magnetometer alignment: corrects for misalignment between the magnetic coordinate frame and the inertial frame.

The figure below illustrates the impact of soft and hard irons on the magnetometers data.



#### 4.4. Sensor fusion algorithm

The core technology behind the AHRS and vertical gyros is the sensor fusion algorithm. Extended Kalman filters are commonly used to perform this data fusion. The following functional block diagram describes the different blocks operating:



At the top level, the 3 axes Gyroscopes are integrated at a high frequency, zero latency to deliver an optimal dynamic response.

##### 4.4.1. Vertical gyro operation

On the other hand, accelerometers are used as vertical reference sensors and are blended through the Kalman filter with the gyro estimated attitude. The Extended Kalman filter enables both estimation of attitude, and correction of gyro bias.

However, the zero-acceleration assumption made to measure vertical reference is not always met. A Quality Control module verifies the quality of accelerometer measurements and takes decision about their usability in the Kalman filter. Moreover, a rigorous motion modeling optimizes the accelerometer usage to fit the typical vehicles characteristics and enable zero attitude glitches in case of high rotation speeds.

The vertical gyro mode of operation enables roll, pitch estimation, and an unreferenced yaw angle.

## 4.4.2. Optional heading measurement

To obtain absolute yaw or heading measurements, an additional sensor input is required. Magnetic sensor is used for this purpose. Prior to its integration into the EKF, magnetic measurements need to be calibrated, and pass through a cutting edge Quality Control module.

Since the Earth's magnetic field is very weak, it's easily disturbed by surrounding elements. This module is essential to assess the quality of the magnetic field by analyzing the measurement strength (norm) and additional parameters. In case of inconsistent measurement, it's key to let the filter operate in its nominal condition, avoiding invalid sensor measurements.

Similarly to the accelerometer integration, our magnetic measurement integration optimizes its response in case of high dynamics.

## 4.4.3. Further optimizations

The sensor fusion algorithm implemented into the Ellipse AHRS also benefits from several innovations:

- **Motion profiles:** Specific error models were implemented to match the typical dynamics of your own platform and enable best performance in your application.
- **Integration of ZARUT:** Applicable in certain motion profiles, the system can detect when the IMU is stationary. The algorithm recognizes and speeds up gyro bias estimation.
- **No gimbal lock:** It is important to note that the Ellipse series operate correctly in all conditions and do not suffer from the well-known Gimbal lock effect, occurring near  $\pm 90^\circ$  of pitch angle. If such orientation should occur, make sure to use quaternions as orientation output.

## 5. Performance assessment

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The proposed algorithm is integrated into the Ellipse firmware v3.x.

To assess its performance, extensive testing was conducted across various environments including the most challenging one for an AHRS. In this section, the result of the testing is presented for the three environments:

- Wind rose: in static with heading changes
- Typical dynamics: in typical marine dynamics with no special maneuvers
- High dynamics: in airborne with both regular and aerobatic maneuvers

### 5.1. Wind rose

#### 5.1.1. Test description

This test aims to evaluate the performance of the Ellipse in a fixed position, free from significant magnetic disturbances, with regular heading changes during the acquisition. The chosen motion profile was selected to avoid having an active ZARUT function, with the goal of challenging the system and assessing the Ellipse's ability to maintain accuracy without relying on this update method. The objective is to simulate near-static applications in marine environments.

#### 5.1.2. Test conditions

During this test, the Ellipse was mounted on an aluminum bar. It was positioned at eight randomly distributed heading angles across a full 360° rotation. Each position was held for 15 minutes to analyze stability over a moderate duration.

The reference for this test was generated using Qinertia by processing the same data as a Tightly Coupled Post Processed solution, using the dual antenna GNSS to determine the heading. This allows having a reference which is significantly more accurate, without requiring misalignment calculation since the same IMU is used.

Prior to testing, a 3D magnetic calibration was performed in the field to account for any disturbances near the Ellipse.



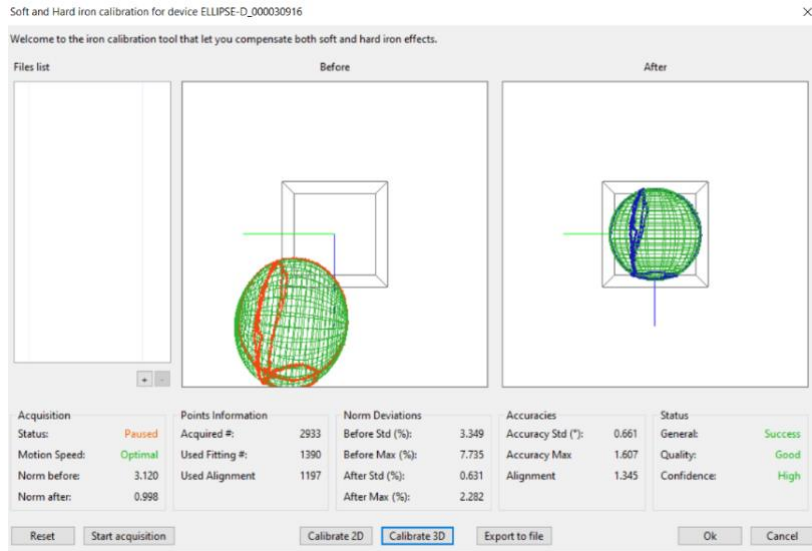


Figure 1: Magnetic calibration results in sbgCenter software

### 5.1.3. Test results

The following results were observed:

	RMS	MAX	Error percentiles		
			68%	95%	99.7%
Roll (°)	0.099	0.296	0.107	0.15	0.212
Pitch (°)	0.085	0.245	0.092	0.126	0.194
Heading (°)	0.737	1.5	0.8	1.336	1.496

The graphs below represent the cumulative distribution function CDF of the error for roll, pitch, and heading.

The overall error distribution follows expected statistical patterns. The 1-sigma error for heading is 0.8°, with the maximum observed error being about 1.5°. Roll and pitch measurements demonstrate good performance, with the 1-sigma error remaining below 0.15°, fully meeting the specified performance requirements of the Ellipse AHRS.

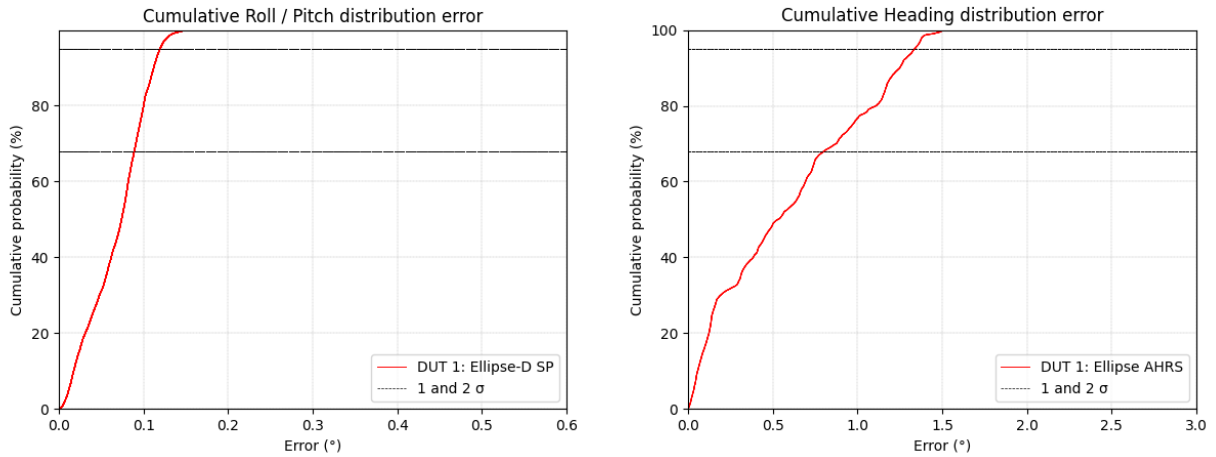


Figure 2: CDF of roll, pitch and heading error - static

To analyze the stability of the errors over time, the graphs below present time series data for roll, pitch, and yaw errors, alongside the standard deviation estimates provided by the Ellipse AHRS. The 2x and 3x standard deviation bands are also included.

Most of the error remains within the one-sigma band, indicating that the standard deviation provides a reliable representation of the error.

We also observe some very small spikes in the standard deviation of roll and pitch during heading changes. However, the overall error remains unaffected by the dynamics of these transitions, demonstrating the robustness of the system in maintaining accuracy even during directional shifts.

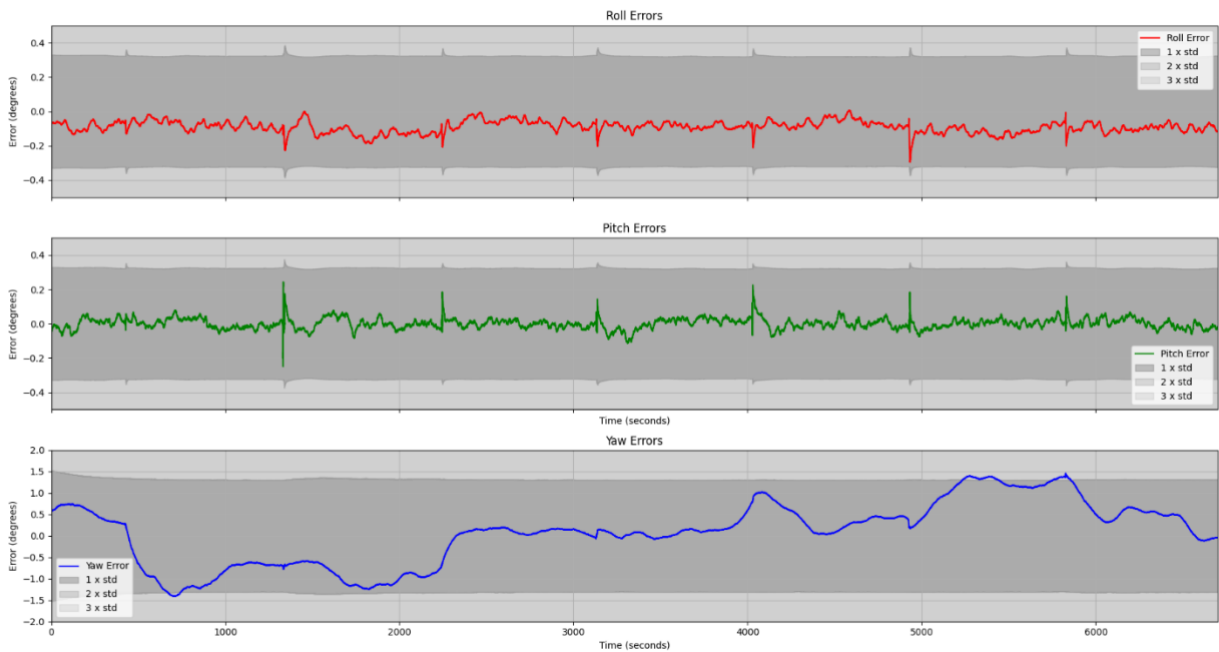


Figure 3: roll, pitch and yaw timeseries error - wind rose

## 5.2. Typical dynamics

### 5.2.1. Test description

The objective of this test was to assess the Ellipse performance in typical marine operations with no specific dynamics.

The TC-PPK solution from the Apogee, a high-end INS with a tactical-grade IMU and a geodetic GNSS receiver, was used as the reference.

### 5.2.2. Test conditions

For this test, data was collected using a small boat in Cagnes-sur-Mer, near Nice. The dataset covers a trajectory of ~1 hour, mainly a long straight-line path, with the vessel maintaining a speed of ~10km/h (5.5knots).

The boat was equipped with various SBG products, including the Ellipse. Although a dual-antenna GNSS model was installed, we used QInertia to disable the GNSS and reprocess the data using only the magnetometers, forcing the AHRS mode.



*Figure 4: Test setup - Typical dynamics*

Once the setup was completed and before the actual acquisition, a 2D magnetic calibration was performed using circular maneuvers. The figure below displays the results of the magnetic calibration in sbgCenter.

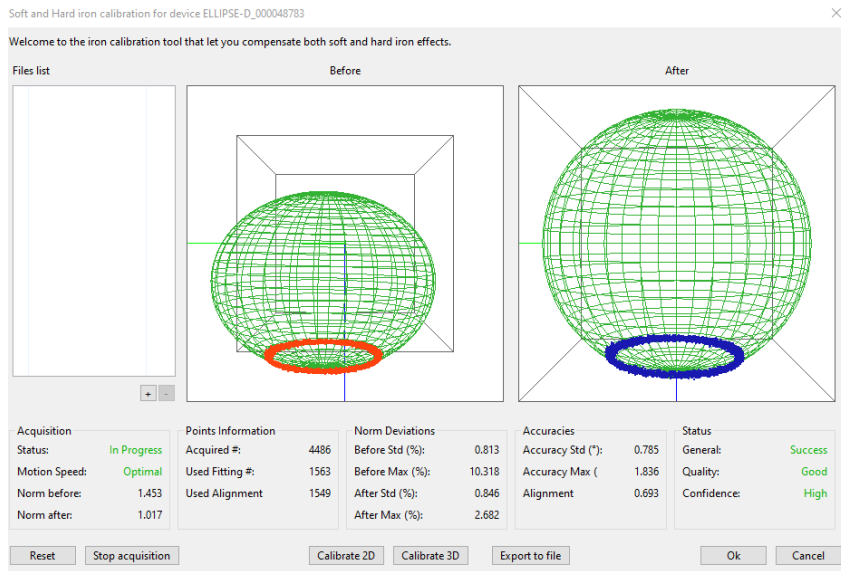


Figure 5: Magnetic calibration results in sbgCenter software

### 5.2.3. Test results

The following results were observed:

	RMS	Max	Error percentiles		
			68%	95%	99.7%
Roll (°)	0.378	0.826	0.415	0.667	0.782
Pitch (°)	0.104	0.335	0.106	0.201	0.295
Heading (°)	0.652	1.281	0.717	1.118	1.272

The graphs below present cumulative distribution functions for the Ellipse AHRS system, showing both roll/pitch errors (left) and heading errors (right).

The roll/pitch measurements maintain good precision with a 1-sigma error of approximately 0.2°. The steep slope of the curve reflects consistent performance and stability, with minimal outliers.

For heading measurements, the 1-sigma error is approximately 0.8° and nearly all heading errors stay below 1.3°.

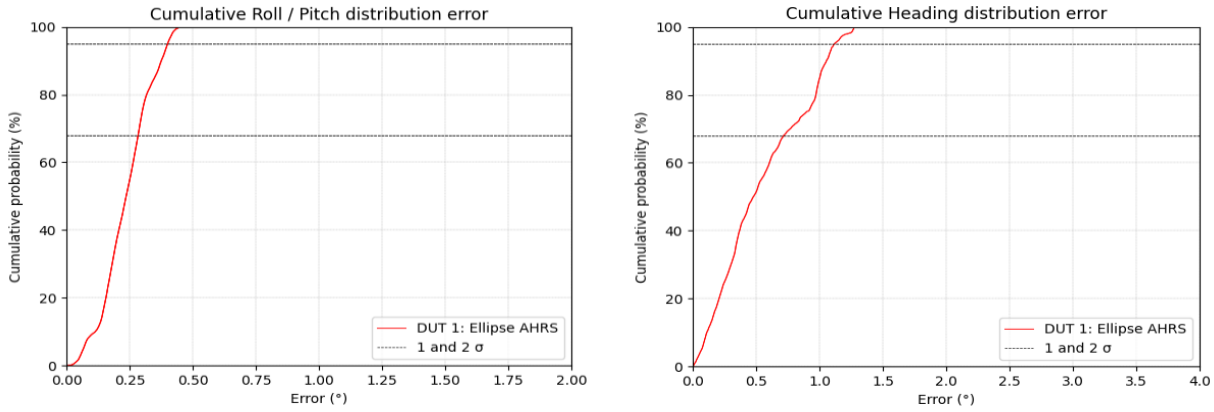


Figure 6: CDF of roll/pitch and heading

The figure 6 displays the error time series for roll, pitch, and yaw, along with the standard deviation as output by the Ellipse. The 2x and 3x standard deviation bands were included too. We observe that most of the error falls within the one-sigma band, which means that the standard deviation represents quite well the error.

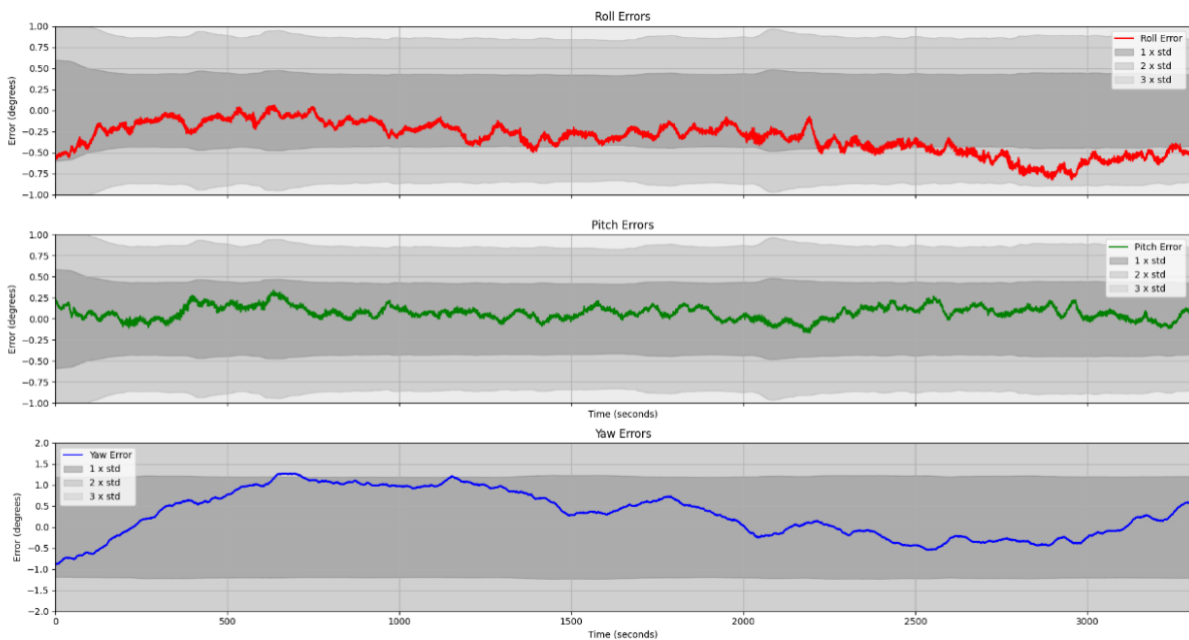


Figure 7: Roll, pitch and yaw error timeseries - Typical dynamics

By plotting the heading error along the trajectory, we observe that the error is particularly high when the vessel approaches the port, without exceeding 1.1°. This shows the system's ability to handle magnetic disturbances effectively.

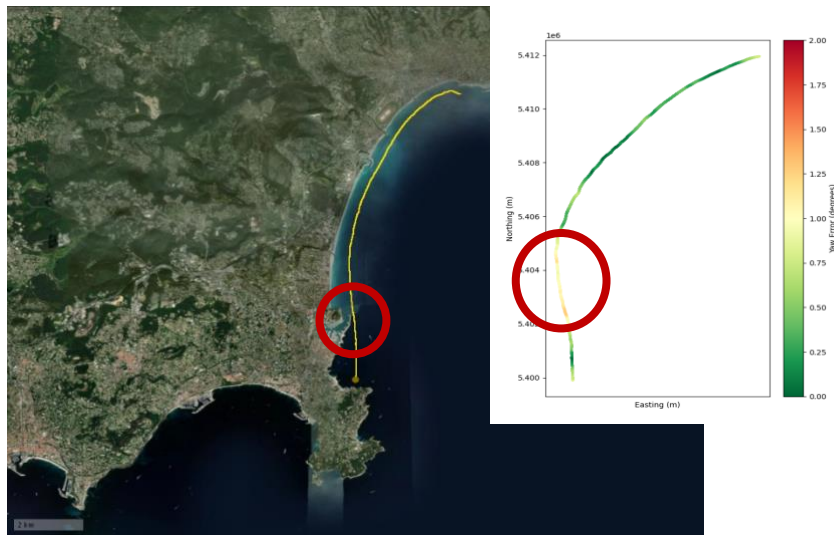


Figure 8: Impact of magnetic disturbances

### 5.3. Challenging environments

#### 5.3.1. Test description

The objective of this test is to evaluate the performance of the Ellipse AHRS in a combination of highly disturbed magnetic environments and high dynamics.

#### 5.3.2. Test conditions

For this test, the Ellipse was subjected to an intense flight campaign in a highly disturbed environment aboard a small training aircraft. The combination of onboard electronics and dynamic maneuvers, such as climb-outs and steep turns, created significant challenges for the system.

Ellipse was mounted alongside other SBG systems products, including the Apogee, high tactical grade INS, which Qinertia TC-PPK solution serves as a reference.



Figure 9: Test setup - challenging environments

Once the setup was completed and prior to the real acquisition, circular maneuvers were conducted to perform a 2D magnetic calibration of the magnetometers.

### 5.3.3. Test results

The figure 9 shows the error time series for roll, pitch, and yaw, along with the standard deviation output by the Ellipse. The 2x and 3x standard deviation bands are also included.

Most errors remain within the one-sigma band, indicating that the standard deviation accurately represents the error. Roll and pitch errors stay below 2°, while heading error slightly exceeds 2°.

These results highlight the system’s ability to maintain correct performance in both highly magnetically disturbed and highly dynamic environments.

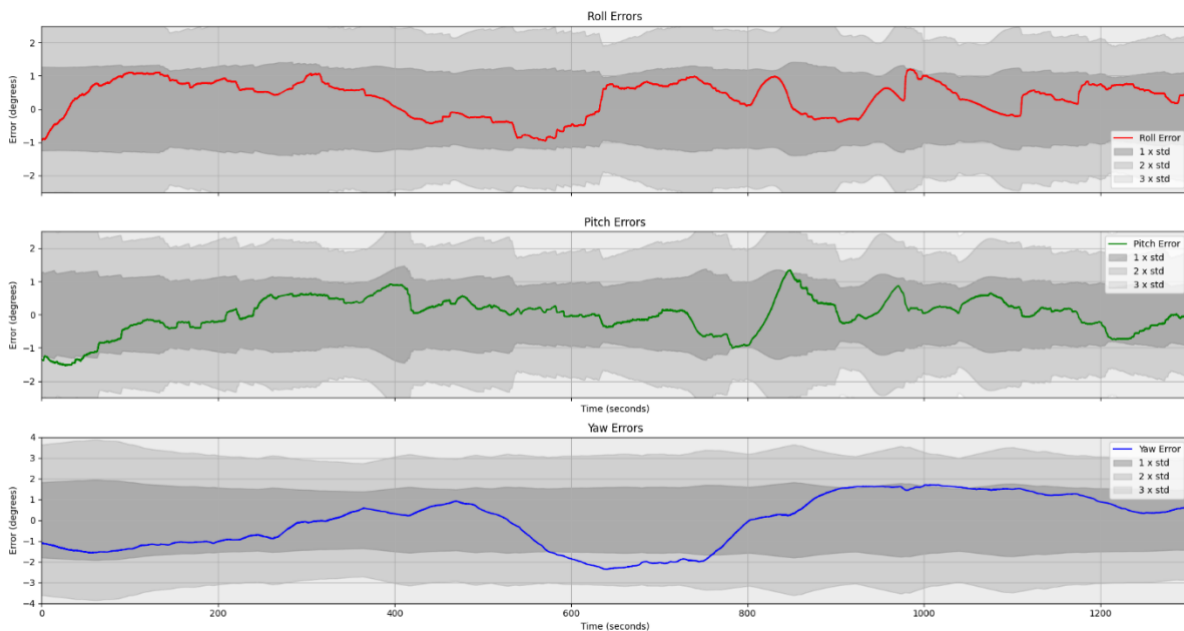


Figure 10: Roll, pitch and yaw error timeseries - High dynamics

This test flight involved numerous aerobatic maneuvers, dynamic movements, and frequent changes in roll and pitch angles. To analyze when the errors occurred and their correlation with the movements, we plotted the roll and pitch angles using a color-coded scheme in figure 10, where the color represents the magnitude of the heading error (in degrees), ranging from green (low error) to red (high error).

We observed that the heading error spikes occurred primarily during movements involving both roll and pitch changes. Despite this, the system rapidly recovered to minimal error levels shortly after.

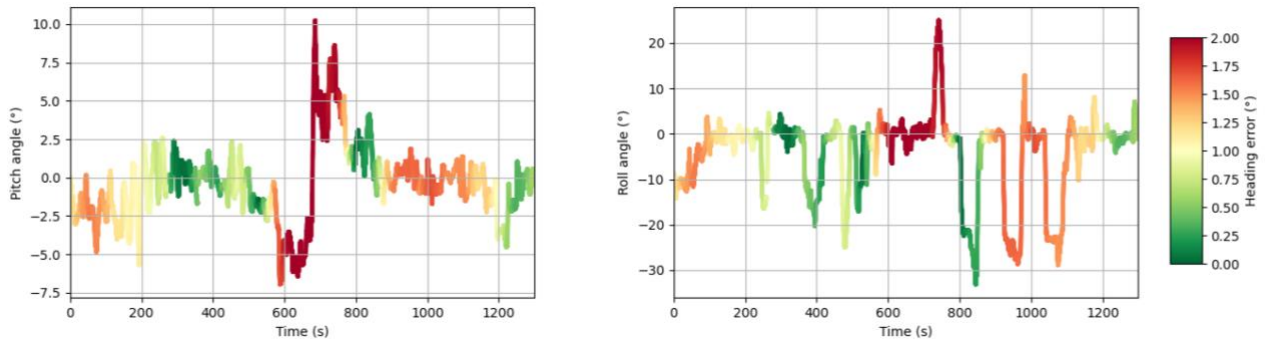


Figure 11: Heading error correlation with roll and pitch changes

From the CDF graphs plotted in figure 11, we observe that the errors in roll and pitch never exceeds 2.5°. The 1-sigma heading error is below 3.

Although this scenario was far from realistic, the test was designed to push the system to its limits. The key takeaway is that the system remains stable, with no significant drift, and the error stays reasonable even under these extreme conditions.

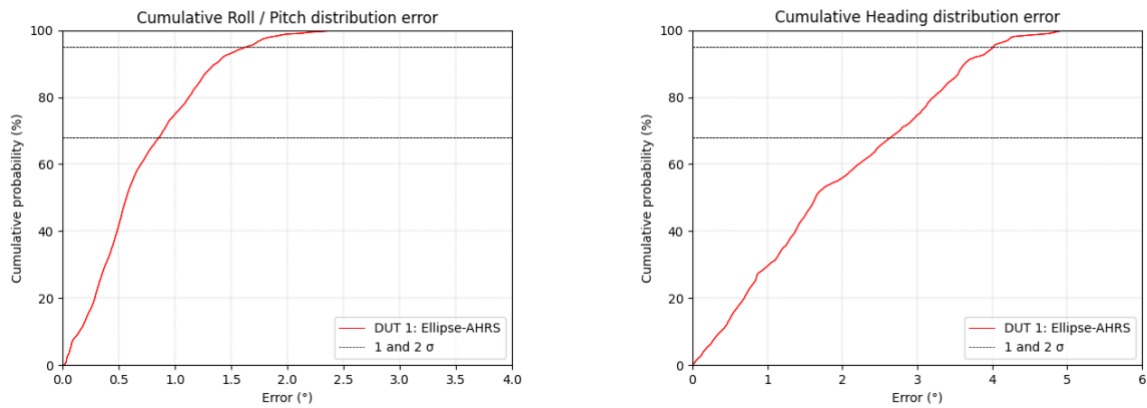


Figure 12: CDF of roll, pitch and heading error angles



## 6. Conclusion

In conclusion, this white paper has outlined the Ellipse's innovative approach to overcoming inherent inertial system biases and introduced a breakthrough algorithm that enhances its performance.

The first section delves into the theoretical foundation of the AHRS algorithm in the Ellipse and explores how the challenges and limitations of MEMS sensors are effectively addressed.

The second part of this paper presents the results of extensive testing of the Ellipse-AHRS in various environments, ranging from quasi-static to highly dynamic and magnetically disturbed conditions. These tests confirm the system's robustness and its ability to provide a reliable solution, even in the most challenging environments.

Finally, the table below summarizes the results and performance across various environments, comparing the achieved performance with the Ellipse specifications.

Measure	Target value (1 $\sigma$ )		Achieved value (1 $\sigma$ )			Status	
	Static	Dynamics	Quasi-static	Typical dynamics	Challenged environment	Static	Typical dynamics
Roll (°)	0.1	0.4	0.1	0.4	0.8	OK	OK
Pitch (°)	0.1	0.4	0.1	0.1	0.8	OK	OK
Heading (°)	0.8	1	0.8	0.7	2	OK	OK

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